SAIM formulas, Nico Stuurman, 11/23/2015

For incident light polarized perpendicular to the plane of incidence, the detected intensity variation at a given pixel in the image relates to the average height of the fluorophore and the angle of incidence as follows:

$$I = A\left(\left|1 + r^{TE} e^{i\phi(H)}\right|\right)^2 + B \quad (1)$$

where
A - accounts for variation in detected intensity due to factors including mean excitation laser intensity, fluorophore density, efficiency of emitted photon detection, etc..
B - is an offset parameter that accounts for background fluorescence in the sample images.
H – is the position above the silicon oxide layer
φ(H) – the phase difference of the direct and reflected light at axial position H given by:

$$\phi(H) = \frac{4\pi}{\lambda}\left(n_b H cos\theta_b\right) \quad (2)$$

r^{TE} – the effective Fresnel coefficient obtained from the transfer matrix m^{TE} according to:

$$r^{TE} = \frac{\left(m_{11}^{TE} + m_{12}^{TE} p_0\right) p_2 + \left(m_{21}^{TE} - m_{22}^{TE} p_0\right)}{\left(m_{11}^{TE} + m_{12}^{TE} p_0\right) p_2 + \left(m_{21}^{TE} + M_{22}^{TE} p_0\right)} \quad (3)$$

where

$$m_{11}^{TE} = \cos\left(k_{0x} d_{ox} \cos_{ox}\right) \quad (4)$$

$$m_{12}^{TE} = \frac{-i}{p_1} \sin\left(k_{0x} d_{ox} \cos\theta_{ox}\right) \quad (5)$$

$$m_{21}^{TE} = -i p_1 \sin\left(k_{0x} d_{ox} \cos\theta_{ox}\right) \quad (6)$$

$$m_{22}^{TE} = \cos\left(k_{0x} d_{ox} \cos\theta_{ox}\right) \quad (7)$$

$$p_0 = n_{Si} \cos\theta_{Si},\ p_1 = n_{ox} \cos\theta_{ox},\ p_2 = n_b \cos\theta_b \quad (8)$$

$$k_i = \frac{2\pi n_i}{\lambda} \quad (9)$$

k_i – wavenumber in the given material
n_{Si} – refractive index of silicon
n_{ox} – refractive index of the oxide layer
n_b – refractive index of the sample
θ_{Si} – angle of incidence in the silicon
θ_{ox} – angle of incidence in the oxide layer
θb– angle of incidence in the sample
By rewriting the effective Fresnel coefficient as a complex number:

$$r^{TE} = c + id \quad (10)$$

Equation 1 can then be rewritten as:

$$I = A\left(1 + 2c\cos\phi(H) - 2d\sin\phi(H) + c^2 + d^2\right) + B \quad (11)$$

Not only can equation 11 be computed about 10 times faster than equation 1, it can also be used to derive partial derivatives needed for Levenberg-Marquard non-linear least square curve fitting, using

$$f = \frac{d}{dH}\phi(H) = \frac{4\pi n_b \cos\theta_b}{\lambda} \quad (12)$$

Partial derivative of equation 11 for H:

$$\frac{d}{dH}(I) = -2Af\left(csin\phi(H) + dcos\phi(H)\right) \quad (13)$$